デジタル表現実習 (140S1017)

ワード（数式）

Proposition 1. Let f be a real-valued function. Then f is convex if and only if $\forall x_1, \ldots, x_n \in X$, $\forall \lambda_1, \ldots, \lambda_n$ such that $\lambda_i > 0$, $\forall i = 1, \ldots, n$, $\sum_{i=1}^{n} \lambda_i = 1$,

$$f(\sum_{i=1}^{n} \lambda_i x_i) \leq \sum_{i=1}^{n} \lambda_i f(x_i).$$

Let X be a topological space. Let $f : X \to (-\infty, \infty]$, $x \in X$, and denote $\mathcal{N}$ the family of all neighborhoods of x. Remember that

$$\liminf_{y \to x} f(y) = \sup_{W \in \mathcal{N}} \inf_{y \in W \setminus \{x\}} f(y).$$

Definition 1. Let $f : X \to (-\infty, \infty]$. f is said to be *lower semicontinuous* if epi f is a closed subset of $X \times \mathbb{R}$. Given $x \in X$, f is said to be *lower semicontinuous at* x if

$$\liminf_{y \to x} f(y) \geq f(x).$$

Example 1. *Determine a curve $x \in C^1[a, b]$ (with $-\infty < a < b < \infty$) with smallest length which connects the two end points (a, x_1) and (b, x_2) (where $x_1, x_2 \in \mathbb{R}$). In other words: We are looking for a minimal point $\bar{x}$ of f on S with*

$$S := \{x \in C^1[a, b] \mid x(a) = x_1 \text{ and } x(b) = x_2\},$$

and

$$f(x) = \int_a^b \sqrt{1 + \dot{x}(t)^2} dt \quad \text{for all } x \in S.$$